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Galen's Logic for Medical Diagnosis after Malatesta's Review: a Development

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1. Summary

Galen, an ancient roman doctor, has given power logical inferences for medical diagnosis by introduction of new polyadic connectives.[1] Malatesta and others have developed modernely these achievements.[2] To manage easily them we present directely in polyadic form a formal theory for standard sentence logic.

2. A Formal Theory $\mathbb{S}$ for Standard Sentence Logic

2.1 *Language*

2.1.1 (),[3] [], *a*, *b*, *c*, … are sentences.
2.1.2 if $\alpha_1, \ldots, \alpha_n$ are sentences then $[\alpha_1\ldots\alpha_n]$ and $\alpha_1\ldots\alpha_n$ are sentences.
2.1.3 Any sentence can only be built by 2.1.1 and 2.1.2.
2.1.4 '$\mapsto$' means 'hence'.

2.2 *Axiom*

2.2.1 ()

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[1] {4.2}, {4.3}

[2] {4.4}, Chapt. VIII

[3] '()' denotes the void expression. To semplify logical calculus rules it has been introduced, exactly as Indians introduced the zero in numerical calculus.

2.3 *Inferences*[4]

2.3.1	$(\ldots[\ldots()\ldots]\ldots)(\ldots[\alpha_1]\ldots)$	$\mapsto$	$(\ldots[\ldots\alpha_1\ldots]\ldots)$
2.3.2	$(\ldots[\alpha_1]\ldots)(\ldots[\ldots()\ldots]\ldots)$	$\mapsto$	$(\ldots[\ldots\alpha_1\ldots]\ldots)$
2.3.3	$[]$	$\mapsto$	$\ldots[]\ldots$
2.3.4	$\ldots[\ldots()\ldots]\ldots\alpha_1\ldots$	$\mapsto$	$\ldots[\ldots\alpha_1\ldots]\ldots\alpha_1\ldots$
2.3.5	$\ldots\alpha_1\ldots[\ldots()\ldots]\ldots$	$\mapsto$	$\ldots\alpha_1\ldots[\ldots\alpha_1\ldots]\ldots$
2.3.6	$\ldots\alpha_1\ldots()\ldots$	$\mapsto$	$\ldots\alpha_1\ldots\alpha_1\ldots$
2.3.7	$\ldots()\ldots\alpha_1\ldots$	$\mapsto$	$\ldots\alpha_1\ldots\alpha_1\ldots$
2.3.8	$\ldots$	$\mapsto$	$[[\ldots]]$

2.4 *Deduction Metatheorem for Sentence Logic*

2.4.1	$[\alpha_1\ldots\alpha_{n\text{-}1}[\alpha_n]]$	$\mapsto$	$\alpha_1, \ldots, \alpha_{n\text{-}1} \mapsto \alpha_n$[5]

2.5 *Abreviations*

2.5.1	$N\alpha_1$	is	$[\alpha_1]$[6]
2.5.2	$\overset{n}{V}\alpha_1\ldots\alpha_n$	is	$()$[7]
2.5.3	$\overset{n}{A}\alpha_1\ldots\alpha_n$	is	$[[\alpha_1]\ldots[\alpha_n]]$[8]
2.5.4	$\overset{n}{Bs}\alpha_1\alpha_2\ldots\alpha_n$	is	$[[\alpha_1]\alpha_2\ldots\alpha_n]$[9]
2.5.5	$\overset{n}{Bc}\alpha_1\alpha_2\alpha_3\ldots\alpha_{n\text{-}1}\alpha_n$	is	$[[\alpha_1]\alpha_2][[\alpha_2]\alpha_3]\ldots[[\alpha_{n\text{-}1}]\alpha_n]$[10]

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[4] Assume that two '…' in corresponding places have the same meaning.

[5] To prove it see {4.5}, Chapt. 1, § 1.4.

[6] '$N\alpha_1$' is the negation of 'α_1'

[7] '$\overset{n}{V}\alpha_1\ldots\alpha_n$' is the *n*-adic tautology.

[8] '$\overset{n}{A}\alpha_1\ldots\alpha_n$' is the *n*-adic logic sum.

[9] See Malatesta's converse sequence implication. {4.1} pp. 13

[10] See Malatesta's converse chain implication. {4.1} pp. 22

2.5.6 $\overset{n}{C}s\alpha_1\alpha_2\ldots\alpha_n$ is $[\alpha_1\alpha_2\ldots[\alpha_n]]$[11]

2.5.7 $\overset{n}{C}c\alpha_1\alpha_2\alpha_3\ldots\alpha_{n\text{-}1}\alpha_n$ is $[\alpha_1[\alpha_2]][\alpha_2[\alpha_3]]\ldots[\alpha_{n\text{-}1}[\alpha_n]]$[12]

2.5.8 $\overset{n}{D}\alpha_1\ldots\alpha_n$ is $[\alpha_1\ldots\alpha_n]$[13]

2.5.9 $\overset{n}{E}\alpha_1\ldots\alpha_n$ is $[[\alpha_1\ldots\alpha_n][[\alpha_1]\ldots[\alpha_n]]]$[14]

2.5.10 $\overset{n}{F}\alpha_1\ldots\alpha_n$ is $[\alpha_1]$

2.5.11 $\overset{n}{G}\alpha_1\ldots\alpha_n$ is $[\alpha_n]$

2.5.12 $\overset{n}{H}\alpha_1\ldots\alpha_n$ is α_n

2.5.13 $\overset{n}{I}\alpha_1\ldots\alpha_n$ is α_1

2.5.14 $\overset{n}{J}c\alpha_1\alpha_2\ldots\alpha_n$ is $[[\alpha_1[\alpha_2]\ldots[\alpha_n]][[\alpha_1]\alpha_2\ldots[\alpha_n]]\ldots[$ $[\alpha_1][\alpha_2]\ldots\alpha_n]]$[15]

2.5.15 $\overset{n}{J}i\alpha_1\alpha_2\ldots\alpha_n$ is $[[[\alpha_1][\alpha_2]\ldots[\alpha_n]][\alpha_1[\alpha_2]\ldots[\alpha_n]][$ $[\alpha_1]\alpha_2\ldots[\alpha_n]]\ldots[[\alpha_1][\alpha_2]\ldots\alpha_n]]$[16]

2.5.16 $\overset{n}{K}\alpha_1\ldots\alpha_n$ is $\alpha_1\ldots\alpha_n$[17]

2.5.17 $\overset{n}{L}s\alpha_1\alpha_2\ldots\alpha_n$ is $\alpha_1\alpha_2\ldots[\alpha_n]$[18]

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[11] See Malatesta's sequence implication. {4.1} pp. 12

[12] See Malatesta's chain implication. {4.1} pp. 21

[13] '$\overset{n}{D}\alpha_1\ldots\alpha_n$' is the *n*-adic logic sum of negations.

[14] See Malatesta's polyadic equivalence (all arguments are equivalent among them). {4.1} pp. 24

[15] See Galen's complete battle (one and only one of its arguments is true). {4.3} pp. 85-90

[16] See Gellius' incomplete battle (one at most of its arguments is true). {4.3} pp. 85-90

[17] '$\overset{n}{K}\alpha_1\ldots\alpha_n$' is the *n*-adic logic product.

[18] The negation of Malatesta's sequence implication.

2.5.18 $\overset{n}{Lc}\alpha_1\alpha_2\alpha_3\ldots\alpha_{n\text{-}1}\alpha_n$ is $[[\alpha_1[\alpha_2]][\alpha_2[\alpha_3]]\ldots[\alpha_{n\text{-}1}[\alpha_n]]]$[19]

2.5.19 $\overset{n}{Ms}\alpha_1\alpha_2\ldots\alpha_n$ is $[\alpha_1]\alpha_2\ldots\alpha_n$[20]

2.5.20 $\overset{n}{Mc}\alpha_1\alpha_2\alpha_3\ldots\alpha_{n\text{-}1}\alpha_n$ is $[[[\alpha_1]\alpha_2][[\alpha_2]\alpha_3]\ldots[[\alpha_{n\text{-}1}]\alpha_n]]$[21]

2.5.21 $\overset{n}{X}\alpha_1\ldots\alpha_n$ is $[\alpha_1]\ldots[\alpha_n]$[22]

2.5.22 $\overset{n}{O}\alpha_1\ldots\alpha_n$ is $[]$[23]

3. Proofs of Polyadic inferences in ■[24]

3.1 $\overset{n}{Cs}\alpha_1\ldots\alpha_n, \overset{k}{K}\alpha_1\ldots\alpha_k \mapsto \overset{n\text{-}k}{Cs}\alpha_{k+1}\ldots\alpha_n, for\, n \geq 2,\ 1 \leq k < n$[25]

3.1.1	$()$	
3.1.2	$[[]]$	2.3.8
3.1.3	$[[]\alpha_1\ldots\alpha_k\alpha_{k+1}\ldots[\alpha_n]]$	2.3.3
3.1.4	$[[\alpha_1\ldots[\alpha_n]]\alpha_1\ldots\alpha_k\alpha_{k+1}\ldots[\alpha_n]]$	2.3.4
3.1.5	$[[\alpha_1\ldots[\alpha_n]]\alpha_1\ldots\alpha_k[[\alpha_{k+1}\ldots[\alpha_n]]]]$	2.3.8
3.1.6	$[\alpha_1\ldots[\alpha_n]],\ \alpha_1\ldots\alpha_k \mapsto [\alpha_{k+1}\ldots[\alpha_n]]$	2.4.1
3.1.7	$\overset{n}{Cs}\alpha_1\ldots\alpha_n, \alpha_1\ldots\alpha_k \mapsto [\alpha_{k+1}\ldots[\alpha_n]]$	2.5.6
3.1.8	$\overset{n}{Cs}\alpha_1\ldots\alpha_n, \overset{k}{K}\alpha_1\ldots\alpha_k \mapsto [\alpha_{k+1}\ldots[\alpha_n]]$	2.5.16
3.1.9	$\overset{n}{Cs}\alpha_1\ldots\alpha_n, \overset{k}{K}\alpha_1\ldots\alpha_k \mapsto \overset{n\text{-}k}{Cs}\alpha_{k+1}\ldots\alpha_n$	2.5.6

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[19] The negation of Malatesta's chain implication.

[20] The negation of Malatesta's converse sequence implication.

[21] The negation of Malatesta's converse chain implication.

[22] '$\overset{n}{X}\alpha_1\ldots\alpha_n$' is the *n*-adic logic product of negations.

[23] '$\overset{n}{O}\alpha_1\ldots\alpha_n$' is the *n*-adic contradiction.

[24] {4.4}, Chapt. VIII

[25] Polyadic *modus ponendo ponens*.

3.2 $\overset{n}{Cs}\alpha_1\ldots\alpha_n, N\overset{n\text{-}k}{Cs}\alpha_{k+1}\ldots\alpha_n \mapsto N\overset{k}{K}\alpha_1\ldots\alpha_k, for\, n\geq 2, 1\leq k<n$[26]

3.2.1	$()$	
3.2.2	$[[]]$	2.3.8
3.2.3	$[[]\alpha_{k+1}\ldots[\alpha_n]\alpha_1\ldots\alpha_k]$	2.3.3
3.2.4	$[[\alpha_{k+1}\ldots[\alpha_n]]\alpha_{k+1}\ldots[\alpha_n]\alpha_1\ldots\alpha_k]$	2.3.4
3.2.5	$[[\alpha_1\ldots[\alpha_n]]\alpha_{k+1}\ldots[\alpha_n]\alpha_1\ldots\alpha_k]$	2.3.4
3.2.6	$[[\alpha_1\ldots[\alpha_n]][[\alpha_{k+1}\ldots[\alpha_n]]][[\alpha_1\ldots\alpha_k]]]$	2.3.8
3.2.7	$[\alpha_1\ldots[\alpha_n]], [[\alpha_{k+1}\ldots[\alpha_n]]]\mapsto[\alpha_1\ldots\alpha_k]$	2.4.1
3.2.8	$\overset{n}{Cs}\alpha_1\ldots\alpha_n,[[\alpha_{k+1}\ldots[\alpha_n]]]\mapsto[\alpha_1\ldots\alpha_k]$	2.5.6
3.2.9	$\overset{n}{Cs}\alpha_1\ldots\alpha_n,N[\alpha_{k+1}\ldots[\alpha_n]]\mapsto[\alpha_1\ldots\alpha_k]$	2.5.1
3.2.10	$\overset{n}{Cs}\alpha_1\ldots\alpha_n,N\overset{n\text{-}k}{Cs}\alpha_{k+1}\ldots\alpha_n\mapsto[\alpha_1\ldots\alpha_k]$	2.5.6
3.2.11	$\overset{n}{Cs}\alpha_1\ldots\alpha_n,N\overset{n\text{-}k}{Cs}\alpha_{k+1}\ldots\alpha_n\mapsto N(\alpha_1\ldots\alpha_k)$	2.5.1
3.2.12	$\overset{n}{Cs}\alpha_1\ldots\alpha_n,N\overset{n\text{-}k}{Cs}\alpha_{k+1}\ldots\alpha_n\mapsto N\overset{k}{K}\alpha_1\ldots\alpha_k$	2.5.16

3.3 $\overset{n}{Ji}\alpha_1\ldots\alpha_n,\alpha_k\mapsto\overset{n\text{-}1}{X}\alpha_1\ldots\alpha_{k\text{-}1}\alpha_{k+1}\ldots\alpha_n, for\, n\geq 2, 1\leq k\leq n$[27]

3.3.1	$()$	
3.3.2	$[[]][[]]\ldots[[]]\ldots[[]]$	2.3.8
3.3.3	$[[\alpha_1]\ldots[]\ldots[\alpha_n]\alpha_k[]][\alpha_1\ldots[]\ldots[\alpha_n]\alpha_k[[\alpha_1]]]\ldots[[\alpha_1]\ldots\alpha_k\ldots$ $[\alpha_n]\alpha_k[]]\ldots[[\alpha_1]\ldots[]\ldots\alpha_n\alpha_k[[\alpha_n]]]$	2.3.3
3.3.4	$[[\alpha_1]\ldots[\alpha_k]\ldots[\alpha_n]\alpha_k[]][\alpha_1\ldots[\alpha_k]\ldots[\alpha_n]\alpha_k[[\alpha_1]]]\ldots[[\alpha_1]\ldots$ $\alpha_k\ldots[\alpha_n]\alpha_k[]]\ldots[[\alpha_1]\ldots[\alpha_k]\ldots\alpha_n\alpha_k[[\alpha_n]]]$	2.3.4
3.3.5	$[[\alpha_1]\ldots[\alpha_k]\ldots[\alpha_n]\alpha_k[[\alpha_1]\ldots[\alpha_{k\text{-}1}][\alpha_{k+1}]\ldots[\alpha_n]]][\alpha_1\ldots[\alpha_k]$ $\ldots[\alpha_n]\alpha_k[[\alpha_1]\ldots[\alpha_{k\text{-}1}][\alpha_{k+1}]\ldots[\alpha_n]]]\ldots[[\alpha_1]\ldots\alpha_k\ldots[\alpha_n]$ $\alpha_k[[\alpha_1]\ldots[\alpha_{k\text{-}1}][\alpha_{k+1}]\ldots[\alpha_n]]]\ldots[[\alpha_1]\ldots[\alpha_k]\ldots\alpha_n\alpha_k[[\alpha_1]$ $\ldots[\alpha_{k\text{-}1}][\alpha_{k+1}]\ldots[\alpha_n]]]$	2.3.5

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[26] Polyadic *modus tollendo tollens*.

[27] Polyadic *modus ponendo tollens I*.

3.3.6 $[[[[\alpha_1]\ldots[\alpha_k]\ldots[\alpha_n]]]\alpha_k[[\alpha_1]\ldots[\alpha_{k-1}][\alpha_{k+1}]\ldots[\alpha_n]]][[[\alpha_1$
$\ldots[\alpha_k]\ldots[\alpha_n]]]\alpha_k[[\alpha_1]\ldots[\alpha_{k-1}][\alpha_{k+1}]\ldots[\alpha_n]]]\ldots[[[[\alpha_1]\ldots$
$\alpha_k\ldots[\alpha_n]]]\alpha_k[[\alpha_1]\ldots[\alpha_{k-1}][\alpha_{k+1}]\ldots[\alpha_n]]]\ldots[[[[\alpha_1]\ldots[\alpha_k]$
$\ldots\alpha_n]]\alpha_k[[\alpha_1]\ldots[\alpha_{k-1}][\alpha_{k+1}]\ldots[\alpha_n]]]$ 2.3.8

3.3.7 $[[[[\alpha_1]\ldots[\alpha_k]\ldots[\alpha_n]][\alpha_1\ldots[\alpha_k]\ldots[\alpha_n]]\ldots[[\alpha_1]\ldots\alpha_k\ldots[\alpha_n]]$
$\ldots[[\alpha_1]\ \ldots[\alpha_k]\ldots\alpha_n]]\alpha_k[[\alpha_1]\ldots[\alpha_{k-1}][\alpha_{k+1}]\ldots[\alpha_n]]]$
2.3.2

3.3.8 $[[[\alpha_1]\ldots[\alpha_k]\ldots[\alpha_n]][\alpha_1\ldots[\alpha_k]\ldots[\alpha_n]]\ldots[[\alpha_1]\ldots\alpha_k\ldots[\alpha_n]]$
$\ldots[[\alpha_1]\ \ldots[\alpha_k]\ldots\alpha_n]],\alpha_k\mapsto[\alpha_1]\ldots[\alpha_{k-1}][\alpha_{k+1}]\ldots[\alpha_n]$
2.4.1

3.3.9 $\overset{n}{J}_i\alpha_1\ldots\alpha_k\ldots\alpha_n,\alpha_k\mapsto[\alpha_1]\ldots[\alpha_{k-1}][\alpha_{k+1}]\ldots[\alpha_n]$ 2.5.15

3.3.10 $\overset{n}{J}_i\alpha_1\ldots\alpha_k\ldots\alpha_n,\alpha_k\mapsto\overset{n-1}{X}\alpha_1\ldots\alpha_{k-1}\alpha_{k+1}\ldots\alpha_n$ 2.5.21

3.4 $\overset{n}{J}_c\alpha_1\ldots\alpha_n,\alpha_k\mapsto\overset{n-1}{X}\alpha_1\ldots\alpha_{k-1}\alpha_{k+1}\ldots\alpha_n, for\, n\geq 2, 1\leq k\leq n$[28]

3.4.1 ()

3.4.2 $[[]]\ldots[[]]\ldots[[]]$ 2.3.8

3.4.3 $[\alpha_1\ldots[]\ldots[\alpha_n]\alpha_k[[\alpha_1]]]\ldots[[\alpha_1]\ldots\alpha_k\ldots[\alpha_n]\alpha_k[]]\ldots[[\alpha_1]\ldots[]$
$\ldots\alpha_n\alpha_k[[\alpha_n]]]$ 2.3.3

3.4.4 $[\alpha_1\ldots[\alpha_k]\ldots[\alpha_n]\alpha_k[[\alpha_1]]]\ldots[[\alpha_1]\ldots\alpha_k\ldots[\alpha_n]\alpha_k[]]\ldots[[\alpha_1]\ldots$
$[\alpha_k]\ldots\alpha_n\alpha_k[[\alpha_n]]]$ 2.3.4

3.4.5 $[\alpha_1\ldots[\alpha_k]\ldots[\alpha_n]\alpha_k[[\alpha_1]\ldots[\alpha_{k-1}][\alpha_{k+1}]\ldots[\alpha_n]]]\ldots[[\alpha_1]\ldots$
$\alpha_k\ldots[\alpha_n]\alpha_k[[\alpha_1]\ldots[\alpha_{k-1}][\alpha_{k+1}]\ldots[\alpha_n]]]\ldots[[\alpha_1]\ldots[\alpha_k]\ldots\alpha_n$
$\alpha_k[[\alpha_1]\ldots[\alpha_{k-1}][\alpha_{k+1}]\ldots[\alpha_n]]]$ 2.3.5

3.4.6 $[[[[\alpha_1]\ldots[\alpha_k]\ldots[\alpha_n]]]\alpha_k[[\alpha_1]\ldots[\alpha_{k-1}][\alpha_{k+1}]\ldots[\alpha_n]]]$ $[[[\alpha_1$
$\ldots[\alpha_k]\ldots[\alpha_n]]]\alpha_k[[\alpha_1]\ldots[\alpha_{k-1}][\alpha_{k+1}]\ldots[\alpha_n]]]\ldots[[[[\alpha_1]\ldots$
$\alpha_k\ldots[\alpha_n]]]\alpha_k[[\alpha_1]\ldots[\alpha_{k-1}][\alpha_{k+1}]\ldots[\alpha_n]]]\ldots[[[[\alpha_1]\ldots[\alpha_k]$
$\ldots\alpha_n]]\alpha_k[[\alpha_1]\ldots[\alpha_{k-1}][\alpha_{k+1}]\ldots[\alpha_n]]]$ 2.3.8

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[28] Polyadic *modus ponendo tollens II*.

3.4.7 $[[[\alpha_1\ldots[\alpha_k]\ldots[\alpha_n]]\ldots[[\alpha_1]\ldots\alpha_k\ldots[\alpha_n]]\ldots[[\alpha_1]\ldots[\alpha_k]\ldots\alpha_n]]\alpha_k[[\alpha_1]\ldots[\alpha_{k-1}][\alpha_{k+1}]\ldots[\alpha_n]]]$ 2.3.2

3.4.8 $[[\alpha_1\ldots[\alpha_k]\ldots[\alpha_n]]\ldots[[\alpha_1]\ldots\alpha_k\ldots[\alpha_n]]\ldots[[\alpha_1]\ldots[\alpha_k]\ldots\alpha_n]], \alpha_k\mapsto[\alpha_1]\ldots[\alpha_{k-1}][\alpha_{k+1}]\ldots[\alpha_n]$ 2.4.1

3.4.9 $\overset{n}{J_c}\alpha_1\ldots\alpha_k\ldots\alpha_n, \alpha_k\mapsto[\alpha_1]\ldots[\alpha_{k-1}][\alpha_{k+1}]\ldots[\alpha_n]$ 2.5.14

3.4.10 $\overset{n}{J_c}\alpha_1\ldots\alpha_k\ldots\alpha_n, \alpha_k\mapsto\overset{n-1}{X}\alpha_1\ldots\alpha_{k-1}\alpha_{k+1}\ldots\alpha_n$ 2.5.21

3.5 $\overset{n}{J_c}\alpha_1\ldots\alpha_n, \overset{n-1}{X}\alpha_1\ldots\alpha_{k-1}\alpha_{k+1}\ldots\alpha_n\mapsto\alpha_k$, *for* $n\geq 2, 1\leq k\leq n$[29]

3.5.1 ()

3.5.2 [[]]…[[]]…[[]] 2.3.8

3.5.3 $[\alpha_1\ldots[\alpha_k]\ldots[\alpha_n][[]\ldots[\alpha_{k-1}][\alpha_{k+1}]\ldots[\alpha_n][\alpha_k]]\ldots[[\alpha_1]\ldots\alpha_k\ldots[\alpha_n][[\alpha_1]\ldots[\alpha_{k-1}][\alpha_{k+1}]\ldots[\alpha_n][]]\ldots[[\alpha_1]\ldots[\alpha_k]\ldots\alpha_n[[\alpha_1]\ldots[\alpha_{k-1}][\alpha_{k+1}]\ldots[][\alpha_k]]$ 2.3.3

3.5.4 $[\alpha_1\ldots[\alpha_k]\ldots[\alpha_n][[\alpha_1]\ldots[\alpha_{k-1}][\alpha_{k+1}]\ldots[\alpha_n][\alpha_k]]\ldots[[\alpha_1]\ldots\alpha_k\ldots[\alpha_n][[\alpha_1]\ldots[\alpha_{k-1}][\alpha_{k+1}]\ldots[\alpha_n][\alpha_k]]\ldots[[\alpha_1]\ldots[\alpha_k]\ldots\alpha_n[[\alpha_1]\ldots[\alpha_{k-1}][\alpha_{k+1}]\ldots[\alpha_n][\alpha_k]]$ 2.3.5

3.5.5 $[[[\alpha_1\ldots[\alpha_k]\ldots[\alpha_n]]][[\alpha_1]\ldots[\alpha_{k-1}][\alpha_{k+1}]\ldots[\alpha_n][\alpha_k]]\ldots[[[[\alpha_1]\ldots\alpha_k\ldots[\alpha_n]]][[\alpha_1]\ldots[\alpha_{k-1}][\alpha_{k+1}]\ldots[\alpha_n][\alpha_k]]\ldots[[[[\alpha_1]\ldots[\alpha_k]\ldots\alpha_n]][[\alpha_1]\ldots[\alpha_{k-1}][\alpha_{k+1}]\ldots[\alpha_n][\alpha_k]]$ 2.3.8

3.5.6 $[[[\alpha_1\ldots[\alpha_k]\ldots[\alpha_n]]\ldots[[\alpha_1]\ldots\alpha_k\ldots[\alpha_n]]\ldots[[\alpha_1]\ldots[\alpha_k]\ldots\alpha_n]][[\alpha_1]\ldots[\alpha_{k-1}][\alpha_{k+1}]\ldots[\alpha_n][\alpha_k]]$ 2.3.2

3.5.7 $[[\alpha_1\ldots[\alpha_k]\ldots[\alpha_n]]\ldots[[\alpha_1]\ldots\alpha_k\ldots[\alpha_n]]\ldots[[\alpha_1]\ldots[\alpha_k]\ldots\alpha_n]], [\alpha_1]\ldots[\alpha_{k-1}][\alpha_{k+1}]\ldots[\alpha_n]\mapsto\alpha_k$ 2.4.1

3.5.8 $\overset{n}{J_c}\alpha_1\ldots\alpha_k\ldots\alpha_n, [\alpha_1]\ldots[\alpha_{k-1}][\alpha_{k+1}]\ldots[\alpha_n]\mapsto\alpha_k$ 2.5.14

3.5.9 $\overset{n}{J_c}\alpha_1\ldots\alpha_k\ldots\alpha_n, \overset{n-1}{X}\alpha_1\ldots\alpha_{k-1}\alpha_{k+1}\ldots\alpha_n\mapsto\alpha_k$ 2.5.21

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[29] Polyadic *modus tollendo ponens*.

3.6 $\overset{n}{A}\alpha_1\ldots\alpha_n, \overset{k}{X}\alpha_1\ldots\alpha_k \mapsto \overset{n\text{-}k}{A}\alpha_{k+1}\ldots\alpha_n, \textit{for}\, n\geq 2, 1\leq k<n$[30]

3.6.1	$()$	
3.6.2	$[[]]$	2.3.8
3.6.3	$[[][\alpha_1]\ldots[\alpha_k][\alpha_{k+1}]\ldots[\alpha_n]]$	2.3.3
3.6.4	$[[[\alpha_1]\ldots[\alpha_n]][\alpha_1]\ldots[\alpha_k][\alpha_{k+1}]\ldots[\alpha_n]]$	2.3.4
3.6.5	$[[[\alpha_1]\ldots[\alpha_n]][\alpha_1]\ldots[\alpha_k][[[\alpha_{k+1}]\ldots[\alpha_n]]]]$	2.3.8
3.6.6	$[[\alpha_1]\ldots[\alpha_n]], [\alpha_1]\ldots[\alpha_k]\mapsto[[\alpha_{k+1}]\ldots[\alpha_n]]$	2.4.1
3.6.7	$\overset{n}{A}\alpha_1\ldots\alpha_n,[\alpha_1]\ldots[\alpha_k]\mapsto[[\alpha_{k+1}]\ldots[\alpha_n]]$	2.5.3
3.6.8	$\overset{n}{A}\alpha_1\ldots\alpha_n, \overset{k}{X}\alpha_1\ldots\alpha_k\mapsto[[\alpha_{k+1}]\ldots[\alpha_n]]$	2.5.21
3.6.9	$\overset{n}{A}\alpha_1\ldots\alpha_n, \overset{k}{X}\alpha_1\ldots\alpha_k\mapsto \overset{n\text{-}k}{A}\alpha_{k+1}\ldots\alpha_n$	2.5.3

4. References

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