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On D'Amelio's Logical Ratio  
and Rules: Consequences in  
Psychiatry

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# On D'Amelio's Logical Ratio and Rules: Consequences in Psychiatry

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## 1. *Introduction*

The study of the rules, which allow the total elaboration of a logical formula, has had a very important contribution by Carlo D'Amelio {D'Amelio (1988)}. By the rules which he has found we can obtain also a better management of the Boolean model of the psychological relations described in the Bari University {De Giacomo, Silvestri and coop. (1976); (1978)}.

In this paper we propose some graphic modifications in the D'Amelio's algebraic-logical language to minimize the mistakes which can happen for the apparent likelihood of the D'Amelio's logical formulas with some algebraic expressions of standard algebra. As D'Amelio, we shall utilize Lukasiewicz's and the Hermes-Scholz's symbolic languages. Also for us the terms "functor" and "operator" are synonyms of "connective". Besides by using the adjectives "logic" and "algebraic" we shall refer to the Boole's and to the standard algebra respectively. Hence, we shall examine some cases of the comprehension of the psychological relations in Boolean representation by the D'Amelio's rules.

This work will be divided in two partes, that will be - if we borrow the Schröder's or Carnap's language - (I.) of pure logic (*reine Logik*), (II.) of applied logic (*angewandte Logik*) respectively.

# I. D'AMELIO'S LOGICAL RATIO AND RULES

## 2. *D'Amelio's Logical ratio*

D'Amelio defines *logical ratio* of p to q, the ratio

$$\left( \frac{p}{q} \right)$$

where the brackets indicate that the logical ratio of p to q has to be considered as an inscindible term; D'Amelio omitts bracketts when they are not strictly necessary. The D'Amelio's logical ratio is characterized by the following features:

- 1) If numerator and denominator are equal, then the ratio value is 1.
- 2) If numerator and denominator are not equal, then the ratio value is equal to numerator.

It results

$$\frac{1}{1} = 1 \quad \frac{0}{1} = 0 \quad \frac{1}{0} = 1 \quad \frac{0}{0} = 1$$

The D'Amelio's logical ratio has, hence, remarkable differences with the algebraic ratio. We can see easily that features of a such logical ratio define a truth table like the logical converse implication (the Lukasiewicz's connective B and the Hermes-Scholz's connective  $\leftarrow$ ). But it has in the D'Amelio's system a calculation priority very higher either than the converse implication in the Lukasiewicz's and Hermes-Scholz's systems, or than the same algebraic ratio. Also, D'Amelio uses the logical ratio to define the following connectives: on the other hand the negation (Np in Lukasiewicz,  $\neg p$  in Hermes-Scholz,  $\frac{0}{p}$  in D'Amelio),

which has a highest calculation priority; on the other hand the material implication ( $Cpq$  in Łukasiewicz,  $p \rightarrow q$  in Hermes-Scholz,  $\frac{q}{p}$  in D'Amelio) and the material converse implication ( $Bpq$  in Łukasiewicz,  $p \leftarrow q$  in Hermes-Scholz,  $\frac{p}{q}$  in D'Amelio), which have a very lower calculation priority. Which has to be the calculation priority of the logical ratio? If we attribute to the logical ratio the calculation priority of "not", then we cannot respect the standard calculation priority of the "implies" and the "conversely implies", and vice versa. It is true that we can use the brackets opportunely, but also in this way the inattention mistakes are very frequent.

We observe also that the truth table of the logical ratio is given if and only if the numerator and the denominator are equal to 0 or to 1; this fact puts a great distinction between the definition of algebraic and D'Amelio's logical sum and product, which have, however, the same symbols + and \*, and this one is another font of inattention mistakes.

We observe, finally, that the truth table of the logical ratio is different in two cases from the algebraic ratio (another source of inattention mistakes) and therefore it is opportune to ask ourselves if there is an algebraic operation more similar to logical "ratio" than the algebraic ratio.

### 3. *The logical power*

To solve the previous problems, we consider the algebraic operation of the power; we have when  $n$ ,  $m$  and  $k$  are positive ( $>0$ ) numbers:

$$0^n=0 \quad n^0=1 \quad n^m=k \quad 0^0=\text{indetermined.}$$

However, we observe that in many computer software (consider, for example, the GWBASIC® in the operating system MSDOS®) we have that  $0^0=1$  because 1 is the result of the more usual algorithms for the calculation of the power when the exponent is 0. Also, in the software, generally, the symbol which denotes the power is  $\hat{\phantom{n}}$ , i. e.  $n^m$  is written  $n\hat{m}$ . Therefore we can put a distinction between the *algebraic power*:

$$0^n=0 \quad n^0=1 \quad n^m=k \quad 0^0=\text{indetermined.}$$

and the *logical power*:

$$0\hat{n}=0 \quad n\hat{0}=1 \quad n\hat{m}=k \quad 0\hat{0}=1$$

which is, hence, the algebraic operation which generally the software uses when it has to calculate a power. As the algebraic power, the logic power has obviously the highest calculation priority among the arithmetical operations.

We can use the logical power to define the logical connectives instead of the logical ratio with lower likelihood that inattention mistakes happen because in this way every Boolean expression can be directly transformed in an algebraic expression for which not only the D'Amelio's rules are valid, but also almost all the algebraic rules are valid. We shall follow the D'Amelio's paper with some changes.

#### 4. *About functors meaning*

##### 4.a) Functor "NOT" or "N" or "¬": negation

Lukasiewicz's language:  $Np$ ;

Hermes-Scholz's language:  $\neg p$ ;

D'Amelio's language:  $\left(\frac{0}{p}\right);$   
 our language:  $0^{\wedge}(p).$

4.b) Functor "INCLUSIVE OR" or "A" or "v": logical sum

Lukasiewicz's language:  $Apq;$   
 Hermes-Scholz's language:  $p \vee q;$   
 D'Amelio's language:  $p+q;$   
 our language:  $0^{\wedge}(0^{\wedge}(p+q)).$

*Observation 4.b.1:* If we use  $0^{\wedge}(0^{\wedge}(p+q))$  instead that simply  $p+q$ , then we obtain a compatible formula either with the Boolean rules [in fact  $\neg\neg p=p$ ] or with the algebraic rules (in the standard algebraic calculation, in fact,  $0^{\wedge}(0^{\wedge}(p+q))=0$  if and only if  $p=0$  and  $q=0$ , otherwise  $0^{\wedge}(0^{\wedge}(p+q))=1$ ].

*Observation 4.b.2:* The properties of the logical sum permit to define easily the polyadic logical sum {Malatesta (1989)}. If  $p_1, \dots, p_n$  are Boolean formulas, then we can define as n-adic logical sum the following formula which we show in various languages:

Lukasiewicz's language:  $A \dots A p_1 \dots p_n;$   
 Hermes-Scholz's language:  $p_1 \vee \dots \vee p_n;$   
 Malatesta's language:  $\bigwedge_{i=1}^n p_i;$   
 D'Amelio's language:  $\sum_{i=1}^n p_i;$   
 our language:  $0^{\wedge}(0^{\wedge}(\sum_{i=1}^n p_i)).$

In our language  $\sum_{i=1}^n p_i$  has exactly the properties of the algebraic expression  $\sum_{i=1}^n p_i$ .

#### 4.c) Functor "AND" or "K" or " $\wedge$ ": logical product

Lukasiewicz's language:	$Kpq$ ;
Hermes-Scholz's language:	$p \wedge q$ ;
D'Amelio's language:	$p^*q$ ;
our language:	$p^*q,$ $\hat{0}(\hat{0}(p)+\hat{0}(q)).$

*Observation 4.c.1:* It is not necessary to use  $\hat{0}(\hat{0}(p^*q))$  instead that simply  $p^*q$  because, differently from  $p+q$ , when  $p$  and  $q$  take always only the value 0 or the value 1,  $p$  and  $q$  takes only the values 0 and 1 respectively.

*Observation 4.c.2:* The properties of the logical product permit to define easily the polyadic logical product {Malatesta (1989)}. If  $p_1, \dots, p_n$  are Boolean formulas, then we can define as  $n$ -adic logical product the following formula which we show in various languages:

Lukasiewicz's language:	$K \dots Kp_1 \dots p_n$ ;
Hermes-Scholz's language:	$p_1 \wedge \dots \wedge p_n$ ;
Malatesta's language:	$\prod_{i=1}^n p_i$ ;
D'Amelio's language:	$\prod_{i=1}^n p_i$ ;
our language:	$\prod_{i=1}^n p_i$ ;

$$0^{\wedge}(\sum_{i=1}^n 0^{\wedge}(p_i)).$$

In our language  $\prod_{i=1}^n p_i$  has exactly the properties of

the algebraic expression  $\prod_{i=1}^n p_i$ , i. e.  $p_1 \cdot p_2 \cdot \dots \cdot p_n$ .

Observe also that the principle of lattice duality, which is described largely {Lombardo Radice (1965), Bell - Machover (1977)} and which is valid also in our language, in way that, for example, we have:

$$\prod_{i=1}^n p_i = 0^{\wedge}(\sum_{i=1}^n 0^{\wedge}(p_i)) \text{ and } \sum_{i=1}^n p_i = 0^{\wedge}(\prod_{i=1}^n 0^{\wedge}(p_i)).$$

#### 4.d) Functor "IF ... THEN" or "C" or "→": material implication

Lukasiewicz's language:  $Cpq$ ;

Hermes-Scholz's language:  $p \rightarrow q$ ;

D'Amelio's language:  $\frac{q}{p}$ ;

our language:  $(q)^{\wedge}(p),$   
 $0^{\wedge}(0^{\wedge}(0^{\wedge}(p)+q)),$   
 $0^{\wedge}(p * 0^{\wedge}(q));$

*Observation 4.d.1* : we write  $(q)^{\wedge}(p)$  and not  $q^{\wedge}p$  because the power in  $(q)^{\wedge}(p)$  has to be calculated after the internal functors of  $q$  and  $p$ . The brackets are necessary because the power in the common algebra has a calculation priority very higher than the material implication in the sentence logic.

*Observation 4.d.2*: Malatesta {Malatesta (1989)} defines two polyadic functors which correspond to the material implication. The *sequence implication*

and the *chain implication*. We can define as n-adic sequence implication the following formula which we show in various languages:

Lukasiewicz's language:  $Cp_1 \dots Cp_{n-1} p_n$ ;  
 Hermes-Scholz's language:  $p_1 \rightarrow (\dots \rightarrow (p_{n-1} \rightarrow p_n) \dots)$ ;  
 Malatesta's language:  $\bigcap_{i=1}^n p_i$ ;  
 our language:  $(\dots ((p_n)^\wedge (p_{n-1}))^\wedge \dots)^\wedge (p_1)$ .

A very known property of the algebrical power permits us to write:

$$(\dots ((p_n)^\wedge (p_{n-1}))^\wedge \dots)^\wedge (p_1) = (p_n)^\wedge \left( \prod_{i=1}^{n-1} (p_i) \right).$$

After, we can define as n-adic chain implication the following formula which we show in various languages:

Lukasiewicz's language:  $K \dots K Cp_1 p_2 Cp_2 p_3 \dots$   
 $Cp_{n-1} p_n$ ;  
 Hermes-Scholz's language:  $(p_1 \rightarrow p_2) \wedge (p_2 \rightarrow p_3) \wedge \dots$   
 $\wedge (p_{n-1} \rightarrow p_n)$ ;  
 Malatesta's language:  $\bigcap_{i=1}^n p_i$ ;  
 our language:  $(p_2)^\wedge (p_1) * (p_3)^\wedge (p_2) * \dots$   
 $* (p_n)^\wedge (p_{n-1})$

In our language we can write also:

$$(p_2)^\wedge (p_1) * (p_3)^\wedge (p_2) * \dots * (p_n)^\wedge (p_{n-1}) = \prod_{i=1}^{n-1} (p_{i+1})^\wedge (p_i).$$

4.e) Functor "IS IMPLIED BY" or "B" or " $\leftarrow$ ":  
 converse material implication

Lukasiewicz's language:  $Bpq$ ;  
 Hermes-Scholz's language:  $p \leftarrow q$ ;  
 D'Amelio's language:  $\frac{p}{q}$ ;  
 our language:  $(p) \hat{ } (q)$ ,  
 $0 \hat{ } 0 \hat{ } (p + 0 \hat{ } q)$ ,  
 $0 \hat{ } (0 \hat{ } p * q)$ ;

*Observation 4.e.1* : we write  $(p) \hat{ } (q)$  and not  $p \hat{ } q$  for the same reasons of the material implication.

*Observation 4.e.2* : Malatesta {Malatesta (1989)} defines two polyadic functors which correspond to the converse material implication. The converse sequence implication and the converse chain implication. We can define as n-adic converse sequence implication the following formula which we show in various languages:

Lukasiewicz's language:  $B \dots Bp_1 \dots p_n$ ;  
 Hermes-Scholz's language:  $(\dots (p_1 \leftarrow p_2) \dots \leftarrow p_n)$ ;  
 Malatesta's language:  $\overset{n}{B}_s p_i$ ;  
 our language:  $(\dots ((p_1) \hat{ } (p_2)) \hat{ } \dots) \hat{ } (p_n)$ .

A very known property of the algebrical power permitts us to write:

$$(\dots ((p_1) \hat{ } (p_2)) \hat{ } \dots) \hat{ } (p_n) = (p_1) \hat{ } \left( \prod_{i=1}^{n-1} (p_{i+1}) \right).$$

After, we can define as n-adic converse chain implication the following formula which we show in various languages:

Lukasiewicz's language:	$K...KBp_1p_2Bp_2p_3...$ $Bp_{n-1}p_n;$
Hermes-Scholz's language:	$(p_1 \leftarrow p_2) \wedge (p_2 \leftarrow p_3) \wedge ...$ $\wedge (p_{n-1} \leftarrow p_n);$
Malatesta's language:	$\prod_{i=1}^n B_c p_i;$
our language:	$(p_1)^\wedge (p_2)^*(p_2)^\wedge (p_3)^* ...$ $*(p_{n-1})^\wedge (p_n)$

In our language we can write also:

$$(p_1)^\wedge (p_2)^*(p_2)^\wedge (p_3)^* ... *(p_{n-1})^\wedge (p_n) = \prod_{i=1}^{n-1} (p_i)^\wedge (p_{i+1}).$$

4.f) Functor "IF AND ONLY IF" or "E" or " $\leftrightarrow$ ":  
material equivalence

Lukasiewicz's language:	$E p q;$
Hermes-Scholz's language:	$p \leftrightarrow q;$
D'Amelio's language:	$\left[ \frac{p}{q} \right]^* \left[ \frac{q}{p} \right];$
our language:	$(p)^\wedge (q)^*(q)^\wedge (p);$

*Observation 4.f.1* : Malatesta {Malatesta (1989)} defines a polyadic functor which corresponds to the material equivalence. We can define as n-adic material equivalence the following formula which we show in various languages:

Lukasiewicz's language:	$K...KEp_1p_2Ep_2p_3...$ $Bp_{n-1}p_n;$
Hermes-Scholz's language:	$(p_1 \leftrightarrow p_2) \wedge (p_2 \leftrightarrow p_3) \wedge ...$ $\wedge (p_{n-1} \leftrightarrow p_n);$
Malatesta's language:	$\prod_{i=1}^n E p_i;$

our language:

$$\begin{aligned}
 & (p_1)^\wedge (p_2)^* (p_2)^\wedge (p_1)^* \\
 & (p_2)^\wedge (p_3)^* (p_3)^\wedge (p_2)^* \\
 & \dots^* (p_{n-1})^\wedge (p_n)^* (p_n)^\wedge ( \\
 & p_{n-1})
 \end{aligned}$$

In our language we can write also:

$$\begin{aligned}
 & (p_1)^\wedge (p_2)^* (p_2)^\wedge (p_1)^* (p_2)^\wedge (p_3)^* (p_3)^\wedge (p_2)^* \dots^* (p_{n-1} \\
 & )^\wedge (p_n)^* (p_n)^\wedge (p_{n-1}) = \prod_{i=1}^{n-1} (p_i)^\wedge (p_{i+1})^* (p_{i+1})^\wedge (p_i).
 \end{aligned}$$

and, for the commutativity of the algebraic product, we can obtain the result:

$$\prod_{i=1}^{n-1} (p_i)^\wedge (p_{i+1})^* (p_{i+1})^\wedge (p_i) = \prod_{i=1}^{n-1} (p_i)^\wedge (p_{i+1})^* \prod_{i=1}^{n-1} (p_{i+1})^\wedge (p_i)$$

which is identical to a result which has been obtained by Malatesta {Malatesta (1989)}. In the Malatesta's language we write it:

$$\text{EK} \prod_{i=1}^n C_{p_i} \prod_{i=1}^n B_{c_{p_i}} \prod_{i=1}^n E_{p_i}$$

## 5. *The D'Amelio's rules*

D'Amelio has put five rules to obtain the total elaboration of a logical formula as it happens in the standard algebra {D'Amelio (1988)}. These rules are valid also replacing the D'Amelio's logical ratio with our logical power because the two operations have the same features, in fact we have:

$$\frac{1}{1} = 1 \quad \frac{0}{1} = 0 \quad \frac{1}{0} = 1 \quad \frac{0}{0} = 1$$

and

$$1 \hat{1} = 1 \quad 0 \hat{1} = 0 \quad 1 \hat{0} = 1 \quad 0 \hat{0} = 1.$$

Therefore we can write five valid correspondent rules using the logic power instead of the logic ratio. Their demonstrations are identical to demonstrations which have been given by D'Amelio.

Let  $X(1) \dots X(n) \dots$  and  $Y(1) \dots Y(n) \dots$  be any Boolean well formed formulas, let  $\square$  and  $\#$  be either  $+$  or  $*$ , let  $\gg$  be the symbol of the univocal inference and  $=$  the symbol of the biunivocal inference, we have:

5.a) Zero rule:  $X(n) \gg 0 \hat{0} X(n)$ ; if  $X(n)$  is not a logical sum, then  $0 \hat{0} X(n) \gg X(n)$

*Observation 5.a.1* : the D'Amelio's zero rule is simply:

$$X(n) = \frac{0}{\frac{0}{X(n)}}$$

but we have observed previously that  $0 \hat{(0 \hat{\sum}_{i=1}^n (p_i))}$

is always equal to 1 or to 0, but  $\sum_{i=1}^n p_i$  can be equal

to an other number when we want conserve the complete identity of properties between the logical sum "+" and the Boolean sum "+". However, this power loss of the zero rule is only apparent. In

fact we can substitute always  $\sum_{i=1}^n p_i$  with

$0 \hat{(\prod_{i=1}^n 0 \hat{(p_i)})}$ , as we have shown before, then we

can use the zero rule.

5.b) First rule:  $(X(c) \square X(d)) \hat{=} (Y(a) \# Y(b)) = (X(c)) \hat{=} (Y(a) \# Y(b)) \square (X(d)) \hat{=} (Y(a) \# Y(b))$

5.c) Second rule:  $(X(c) + X(d)) \hat{=} (Y(a) * Y(b)) = (X(c)) \hat{=} (Y(a)) + (X(d)) \hat{=} (Y(b)) + (X(d)) \hat{=} (Y(a)) + (X(d)) \hat{=} (Y(b))$

5.d) Third rule: 
$$\prod_{i=1}^n (X(i)) \hat{=} (Y(i)) = \sum_{m=0}^n \binom{n}{m} \sum_{k=1}^m (a_k * b_k)$$

5.e) Fourth rule: For  $1 \leq m \leq n$  and  $n \leq k$ ,  $\left[ \sum_{i=1}^n X(i) \right] \hat{=} (X(m) * X(k)) = 1$

## 6. *Functor interdefinibility problems in D'Amelio's language*

It is well known that every logical functor is definible by the functor sets: {"AND", "NOT"}, {"OR (inclusive)", "NOT"}, {"IF ... THEN", "NOT"}, {"IS IMPLIED BY", "NOT"} and so on (for a large review on the functor interdefinibility problems see Mendelson {Mendelson (1964)} and Malatesta {Malatesta (1978)}). Therefore in the D'Amelio's language we can use only the logical ratio, the truth values 1 and 0 and the Boolean variables to obtain all the Boolean formulas. In fact, the logical ratio has the same truth table of the functor "IS IMPLIED BY" and every negation is equivalent to the logical ratio which has the truth value 0 as numerator and the argument of the negation as denominator. As the logical power has the same truth table of the logical ratio, we can define all the Lukasiewicz's dyadic sixteen functors also in terms of logical power:

Lukasiew. functors	Their meaning	A logical power definition
Vpq	$p \vee \neg p$	$(p) \hat{=} (p)$
Apq	$p \vee q$	$q \hat{=} (0 \hat{=} (p))$
Bpq	$p \leftarrow q$	$(p) \hat{=} (q)$
Cpq	$p \rightarrow q$	$(q) \hat{=} (p)$
Dpq	$\neg(p \wedge q)$	$0 \hat{=} ((q) \hat{=} (p))$
Epq	$p \leftrightarrow q$	$0 \hat{=} ((0 \hat{=} ((q) \hat{=} (p)))) \hat{=} ((p) \hat{=} (q))$
Fpq	$\neg p$	$0 \hat{=} (p)$
Gpq	$\neg q$	$0 \hat{=} (q)$
Hpq	$q$	$q$
Ipq	$p$	$p$
Jpq	$\neg(p \leftrightarrow q)$	$(0 \hat{=} ((q) \hat{=} (p))) \hat{=} ((p) \hat{=} (q))$
Kpq	$p \wedge q$	$0 \hat{=} ((0 \hat{=} (q)) \hat{=} p)$
Lpq	$p \wedge \neg q$	$0 \hat{=} ((q) \hat{=} (p))$
Mpq	$\neg p \wedge q$	$0 \hat{=} ((p) \hat{=} (q))$
Xpq	$\neg p \wedge \neg q$	$0 \hat{=} ((0 \hat{=} (q)) \hat{=} (p))$
Opq	$p \wedge \neg p$	$0 \hat{=} ((p) \hat{=} (p))$

## II. LOGICAL - ALGEBRAIC APPLICATIONS IN PSYCHIATRY

### 7. *De Giacomo's model on human relations*

It has been proposed a very good Boolean model of the human relations {De Giacomo, Silvestri and coop. (1977) (1978)}.

If  $p$  represents the mental world of a subject,  $q$  represents the mental world of an other subject and  $\mu_n$  (where  $n$  is any integer non negative number) represents any Boolean dyadic functor, then, after a single interaction between the two subjects, we have:

for the first subject:  $p \Rightarrow \mu_1 pq,$

for the second subject:  $q \Rightarrow \mu_2 pq,$

(where the sign  $\Rightarrow$  means "becomes"),

i. e., for example, if the mental world of the first subject is represented by  $p$  before the interaction with the second subject and during this interaction the first subject accepts either all the mental world  $q$  of the second subject or all his mental world, then this one becomes the set union of  $p$  and  $q$  which we can put in correspondence with the Boolean formula  $Apq$ ; so we have that  $\mu_1 = A$ , and hence:

for the first subject:  $p \Rightarrow Apq;$

for the psychiatric interpretation of every dyadic functor see De Giacomo {De Giacomo P., Silvestri A. and coop. (1977) (1978)}, however, we resume:

symmetric interaction field:

O I accept nothing,

K I accept only our agreement,

- L I accept only myself but without our agreement,
- I I accept only myself;

complementary interaction field:

- M I accept only you but without our agreement,
- H I accept only you,
- J I accept myself and you, but not our agreement,
- A I accept myself and you;

antifunctions (generally in psychotical patients):

- X I accept only which is out of me and of you  
(delirium),
- E I accept only the delirium and our agreement,
- G I accept only the delirium, myself, but not our  
agreement,
- B I accept only the delirium and myself,
- F I accept only the delirium, you, but not our  
agreement,
- C I accept only the delirium and yourself,
- D I refuse only our agreement,
- V I accept everything.

We observe that the antifunctions are the symmetric and the complementary interaction, but with delirium.

But, for a subject, to have an important relation with an other subject means to have many single interactions with him. It is possible to evaluate the way in which a subject puts himself in relation with an other subject by the percentual frequency of every dyadic connective which occurs in the interactions between the two subjects. See the FIG. 1.

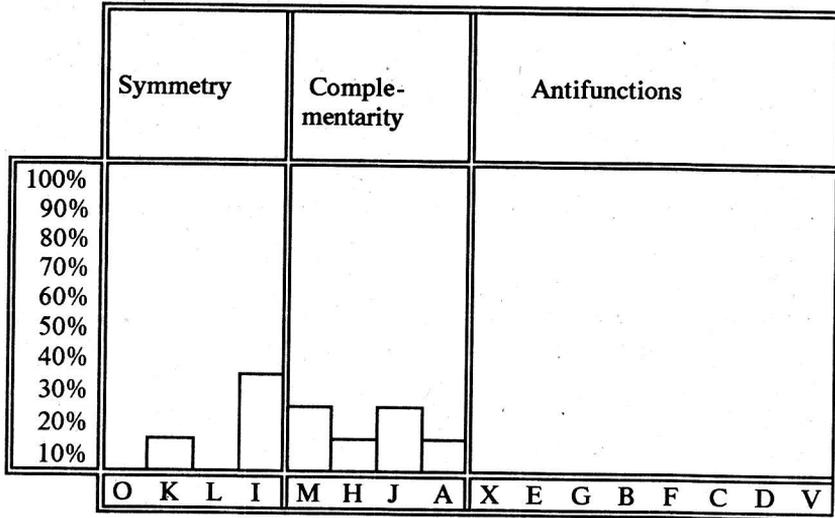


FIG. 1

The FIG. 1 is a whole description of a possible way of a subject to put himself in relation with an other subject. De Giacomo has also built a test to scrutinize the generic way of a subject to put himself in relation with the other men.

Hence, De Giacomo and cooperators state the problem of changement of the way of a subject to put himself in a single interaction with an other subject in function of the way of this last one to put himself in the same interaction. Booleanly we have:

if  
 and  
 for the first subject:  $p \Rightarrow \mu_1 p q$   
 for the second subject:  $q \Rightarrow \mu_2 q p$ ,

then  
 and  
 for the first subject:  $\mu_1 p q \Rightarrow \mu_1 p \mu_2 q p$   
 for the second subject:  $\mu_2 p q \Rightarrow \mu_2 q \mu_1 p q$ .

In general, if  $p$  is the mental world of the first subject,  $\mu_p$  his beginning way of interaction,  $q$  the mental world of the second subject,  $\mu_q$  his beginning way of interaction, then we have for a sequence of close interactions between two subjects (for example, a psychotherapy):

First subject	Second subject	Time
p	q	↓
$\mu_p p q$	$\mu_q q p$	
$\mu_p p \mu_q q p$	$\mu_q q \mu_p p$	
$\mu_p p \mu_q \mu_q q \mu_p p p$	$\mu_q q \mu_p \mu_p p \mu_q q p$	
.....	.....	

De Giacomo and coworkers have built a "paradoxe table" (See FIG. 2). We report it in the Łukasiewicz's language with the convention that the arguments of every functor are, always in this order, p and q:

	O K L I	M H J A	X E G B F C D V
O	O O L I	O O J A	A J V V I L V V
K	O K L I	K K A A	J A D V I I V V
L	O O O I	L L M A	H M C V I L C V
I	O K O I	I I H A	M H F V I I C V
M	O O L I	O M L A	I L B B I J B V
H	O K L I	K H I A	L I G B I A B V
J	O O O I	L J O A	K O E B I J E V
A	O K O I	I A K A	O K X B I A E V
X	O O L I	X X D V	A D A V I G A V
E	O K L I	E E V V	J V J V I B A V
G	O O O I	G G F V	H F H V I G H V
B	O K O I	B B C V	M C M V I B H V
F	O O L I	X F G V	I G I B I D I V
C	O K L I	E C B V	L B L B I V I V
D	O O O I	G D X V	K X K B I D K V
V	O K O I	B V E V	O E O B I V K V

FIG. 2

In FIG. 2: in across there are the beginnig interaction ways of the first subject ( $\mu_1 p q$ ); in down there are the beginnig interaction ways of the second subject ( $\mu_2 p q$ ); in the cells the interaction ways of the first subject after a single interaction with the second subject ( $\mu_1 p \mu_2 p q$ ).

## 8. *The D'Amelio's results in the De Giacomo's model*

In this perspective every Boolean formula can be interpreted as the way of a subject to put himself in interaction with a set of other subjects ( $n-1$  subjects precisely where  $n$  is the number of distinct Boolean variables which occur in the Boolean formula). The five D'Amelio's rules, hence, have to be also psychodynamic rules. As psychodynamics rules the D'Amelio's rules have a very interesting meaning: very complex human relation systems are equivalent to very more simple systems, in fact in many cases the D'Amelio's rules permit to simplify the Boolean formulas (i. e. often by the decrease of the distinct variable number which in psychodynamics corresponds to the subject number). A conclusion which we can obtain from this fact is that in many psychodynamic systems the essential components are much less than those which appear. If we use De Giacomo's model to represent the psychodynamic of any group, then D'Amelio's rules, by the simplification of the Boolean formulas, are very useful to answer at least to these clinical problems of the system setting:

- 1) How many subjects have to be really treated in a psychotherapy on the scrutinized group of subjects? (a psychotherapy on a lesser number of subjects is more easy and convenient).
- 2) What is the importance of the loss of the non-availability of a member for the group therapy? (Does it correspond to a Boolean variable which can be eliminated by the D'Amelio's rules?).
- 3) What is the smaller quantity of resources of time and workers which has to be used to solve a psychodynamic problem? (When we have few resources to project some group psychotherapy, to simplify the relational

structure of the analyzed group can allow a better use of our resources).

The use of the D'Amelio's results in psychiatry is evidently strictly linked to the use in it of Boolean models. The large perspectives which there are in psychiatry for the Boole algebra {A. G. Grappone (1988)} and the mathematical logic in general {I. Matte Blanco (1975)} authorize us to use more and more the D'Amelio's rules.

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