A Formal Theory to Represent in it All the Aristotle's Syllogisms

First Part

Let A , B , C , … be generic sentences.
Let A, B, C, …, A*, B*, C*, …, A', B ', C', …, A* ', B* ', C* ', …, be atomic sentences.

01) Let N A and C A B be well formed formulas (*wffs*).

Put the following axiom outlines:

02) C A C B A ,
03) C C A C B C C C A B C A C ,
04) C C N B N A C C N B A B ,

Put the following inference outline:

05) A , C A B √*f* B (*where* " √*f* " *means* " hence").

06) By 01), …, 05) we can consider N A and C A B as sentences of the standard sentence logic (*SSL*) and we can interpretate:

N A as "not A ",
C A B as "A implies B ".

Therefore we can put some interpreted abbreviations:

07)	V A B	is abbreviation of C A A	(*tautology*),
08)	A A B	is abbreviation of C N A B	(*inclusive or*),
09)	B A B	is abbreviation of C B A	(*converse implication*),
10)	D A B	is abbreviation of A N A N B	(*Sheffer's connective*),
11)	E A B	is abbreviation of N D C A B C B A	(*equivalence*),
12)	F A B	is abbreviation of N A ,	
13)	G A B	is abbreviation of N B ,	
14)	H A B	is abbreviation of B ,	
15)	I A B	is abbreviation of A ,	
16)	J A B	is abbreviation of N E A B	(*exclusive or*),
17)	K A B	is abbreviation of N A N A N B	(*and*),
18)	L A B	is abbreviation of K A N B ,	
19)	MA B	is abbreviation of K N A B ,	
20)	X A B	is abbreviation of K N A N B ,	
21)	O A B	is abbreviation of K A NA	(*contradiction*).

A Formal Structure to Represent in it All the Aristotle's Syllogisms

Second Part

Put the following abbreviations:

22) a A Bis abbreviation of K C AB C A* B,
23) e A Bis abbreviation of X C AB C A* B,
24) i A B is abbreviation of A C AB C A* B,
25) o A B is abbreviation of D C AB C A* B,

26) Let L A be a well formed formula (*wff*).

Put the following abbreviations:

27) M A is abbreviation of N L N A ,
28) Q A is abbreviation of K N L A N L N A .

A Formal Theory to Represent in it All the Aristotle's Syllogisms

Third Part

29) Suppose to have a sentence S in which there are L, M, Q; to obtain an equivalent *SSL* sentence R without them we can execute on S ithe following procedure:

STEP 01: Eliminate every symbol Q by 28).
STEP 02: Eliminate every symbol a, e, i, o by 22), …, 25).
STEP 03: Eliminate every connective V B D E F G H I J L M X O by 07), …, 21).
STEP 04: Replace every A with A' and every A* with A* '.
STEP 05: Replace every L N A with N M A .
STEP 06: Replace every M N A with N L A .
STEP 07: Replace every L A' with K A A' and L A* ' with K A* A* '.
STEP 08: Replace every M A' with A A A' and M A* ' with A A* A* '.
STEP 09: Replace every L K A A' with K AA' and L K A * A* ' with K A * A* '.
STEP 10: Replace every M K A A' with K AA' and M K A * A* ' with K A * A* '.
STEP 11: Replace every L A A A' with A AA' and L A A * A* ' with A A * A* '.
STEP 12: Replace every M A A A' with A AA' and M A A * A* ' with A A * A* '.
STEP 13: If any step among 5, …, 12 has been executed,then go to STEP 5.
STEP 14: Replace every L K A C with K L A L C .
STEP 15: Replace every M K A C with K M A M C when K A C is not contradictory, otherwise with K A C .
STEP 16: Replace every L A A C with A L A L C when A A C is not tautologic, otherwise with A A C .
STEP 17: Replace every M A A C with A M A M C .
STEP 18: Replace every L C A C with C M A L C when A A C is not tautologic, otherwise with C A C .
STEP 19: Replace every M C A C with C L A M C .
STEP 20: If any step among 14, …, 19 has been executed, go to STEP 5.

To kown the proof of validity of this procedure see:

A. G. Grappone
Modal Sentence Logic Formulas as Abbreviations of Standard Sentence Logic Formulas, Metalogicon (1990) **III**, 2, pp. 83-100

A Formal Theory to Represent in it All the Aristotle's Syllogisms

Fourth Part

The proof of the procedure 29) in the cited paper permitts us to affirm that our formal theory is isomorphic to standard sentence logic, hence that there is a procedure to transform every theorem of our structure in an equivalent tautology and vice versa and that we can do the following interpretations:

30) L A means "A is necessary ",
31) M A means "N A is not necessary ",
32) Q A means "A is not necessary and N A is not necessary ".

Also, for this isomorphic relation between our formal theory and , as in standard sentence logic, has to be true the deduction metatheorem (Herbrand, 1930) which we show in this form:

33) $\{A\ , \ldots, B\ , C\ \}\ \surd f\ D$ iff $\{A\ , \ldots, B\ \}\ \surd f\ C\,C\ D$.

The Representation of Aristotle's Syllogisms in our formal theory

First part

The sentences which appear in the Aristotle's syllogism premisses are in this form about:

34) Every A is B,
35) It is necessary that every A is B,
36) It is contingent that every A is B,
37) Some A is B,
38) It is necessary that some A is B,
39) It is contingent that some A is B,
40) No A is B,
41) It is necessary that no A is B,
42) It is contingent that no A is B,
43) Some A is not B,
44) It is necessary that some A is not B,
45) It is contingent that some A is not B.

There is a known representation of them in the standard first order predicative calculus. In the same order of the previous sentences we have respectively in standard language (read ○ as "it is contigent that"):

46) (x)(A(x) … B(x))
47) □ ((x)(A(x) … B(x)))
48) ○ ((x)(A(x) … B(x)))
49) (∃x)(A(x) … B(x))
50) □ ((∃x)(A(x) … B(x)))
51) ○ ((∃x)(A(x) … B(x)))
52) (x)~(A(x) … B(x))
53) □ ((x) ~(A(x) … B(x)))
54) ○ ((x)~(A(x) … B(x)))
55) (∃x) ~(A(x) … B(x))
56) □ ((∃x) ~(A(x) … B(x)))
57) ○ ((∃x) ~(A(x) … B(x)))

The Representation of Aristotle's Syllogisms in our formal theory

Second part

As there are the procedure 29) which eliminates the modal operators $\Box$ and $\bigcirc$ from the sentences in which there are not terms (wffs of sentence modal logic), we must only obtain the reduction of the sentences with terms:

46) $(x)(A(x) \ldots B(x))$,
49) $(\exists x)(A(x) \ldots B(x))$,
52) $(x)\sim(A(x) \ldots B(x))$,
55) $(\exists x)\sim(A(x) \ldots B(x))$,

in sentences whithout terms (wffs of sentence modal logic). Observe that among 46) 49) 52) and 55) there are the following laws (in which the first connective is in Polish notation, the remaining formula, in standard language):

58)	D	$(x)(A(x) \ldots B(x))(x)\sim(A(x) \ldots B(x))$,	
59)	C	$(x)(A(x) \ldots B(x))(\exists x)(A(x) \ldots B(x))$,	
60)	J	$(x)(A(x) \ldots B(x))(\exists x)\sim(A(x) \ldots B(x))$,	
61)	D	$(x)\sim(A(x) \ldots B(x))$	$(x)(A(x) \ldots B(x))$,
62)	J	$(x)\sim(A(x) \ldots B(x))$	$(\exists x)(A(x) \ldots B(x))$,
63)	C	$(x)\sim(A(x) \ldots B(x))$	$(\exists x)\sim(A(x) \ldots B(x))$,
64)	B	$(\exists x)(A(x) \ldots B(x))$	$(x)(A(x) \ldots B(x))$,
65)	J	$(\exists x)(A(x) \ldots B(x))$	$(x)\sim(A(x) \ldots B(x))$,
66)	A	$(\exists x)(A(x) \ldots B(x))$	$(\exists x)\sim(A(x) \ldots B(x))$,
67)	J	$(\exists x)\sim(A(x) \ldots B(x))$	$(x)(A(x) \ldots B(x))$,
68)	B	$(\exists x)\sim(A(x) \ldots B(x))$	$(x)\sim(A(x) \ldots B(x))$,
69)	A	$(\exists x)\sim(A(x) \ldots B(x))$	$(\exists x)(A(x) \ldots B(x))$.

The Representation of Aristotle's Syllogisms in our formal theory

Third part

As the validity of a syllogism depends only from the logic relations among its sentences, then we can replace tidily the sentences 46), 49), 52) and 55) in the syllogisms without loss or gain of validity with every set of four sentences which conserves the logic relations 58), …, 69). So, we can use the following substitution which conserve the logic relations 58), …, 69):

70)	(x)(A(x) … B(x))∅		a A B
71)	(x)~(A(x) … B(x))	∅	e A B
72)	(∃x)(A(x) … B(x))	∅	i A B
73)	(∃x) ~(A(x) … B(x))	∅	o A B

So, every Aristotle's syllogism is representable without using terms.

The propositions 1), …, 73) permitt us to affirm an important conclusion:

74) Every Aristotle's syllogism has a correspondent sentence in the standard sentence logic (*SSL*) such that the syllogism is valid iff the correspondent sentence is a tautology. This sentence can be find by 1), …, 73).

Now, we test, for example, all the syllogisms BARBARA by its correspondig *SSL* sentences. All the Aristotle's syllogisms will be tested in a paper which will be published after.

BARBARA (1)

Both premisses assertoric:

a A B , a C A √ƒ a C B .

C a A B C A C A a C B . for 33),
C K C AB C A* B C K C CA C C* A K C CB C C* B for 22),

C K C A B C A* B C K C CA C C* A K C CB C C* B is a tautology (verify it, for example, with the standard tautology calcule), therefore Barbara with both assertoric premisses is a **valid syllogism**.

Both apodeitic premisses:

L a A B , L a C A √ƒ L a C B .

C L a A B C L A C A L a C B .
for 33),
C L K C AB C A* B C L K C CA C C* A L K C CB C C* B
for 22),
C L K C A ' B ' C A* ' B ' C L K C CA ' C C* ' A ' L K C C ' B ' C C* ' B '
for 29): step 04,
C K L C A ' B ' L C A* ' B ' C K L C C 'A ' L C C* ' A ' K L C C ' B ' L C C* ' B '
for 29): step 14,
C K C M A ' L B ' C M A* ' L B ' C K C M C ' L A ' C M C* ' L A ' K C M C ' L B ' C M C* ' L B '
for 29): step 18,
C K C A AA ' K B B ' C A A*A* ' K B B ' C K C A C C ' K AA ' C A C* C* ' K AA ' KC A C C ' K B B ' CA C* C* ' K B B '
for 29): step 07, 08

C K C A AA ' K B B ' C A A*A* ' K B B ' C K C A C C ' K AA ' C A C* C* ' K AA ' K C A C C ' K B B ' C A C* C* ' K B B ' is a tautology (verify it, for example, with the standard tautology calcule), therefore Barbara with both apodeitic premisses is a **valid syllogism**.

BARBARA (2)

Apodeitic major, assertoric minor
L a A B , a C A √*f* L a C B .

C L a A B C A C A L a C B .
for 33),
C L K C AB C A* B C K C CA C C* A L K C CB C C* B
for 22),
C L K C A ' B ' C A* ' B ' C K C CA ' C C* ' A ' L K C C ' B ' C C* ' B '
for 29): step 04,
C K L C A ' B ' L C A* ' B ' C K C C 'A ' C C* ' A ' K L C C ' B ' L C C* ' B '
for 29): step 14,
C K C M A ' L B ' C M A* ' L B ' C K C C ' A ' C C* ' A ' K C M C ' L B ' C M C* ' L B '
for 29): step 18,
C K C A AA ' K B B ' C A A*A* ' K B B ' C K C C ' A ' C C* ' A ' K C A C C ' K B B ' C A C* C* ' K B B '

for 29): step 07, 08

C K C A AA ' K B B ' C A A*A* ' K B B ' C K C C ' A ' C C* ' A ' K C A C C ' K B B ' C A C* C* ' K B B ' is a tautology (verify it, for example, with the standard tautology calcule), therefore Barbara with apodeitic major premisse and assertoric minor premisse is a **valid syllogism**.

Assertoric major, apodeitic minor:
a A B , L a C A √*f* L a C B .

C a A B C L A C A L a C B
for 33),
C K C AB C A* B C L K C CA C C* A L K C CB C C* B
for 22),
C K C A ' B ' C A* ' B ' C L K C CA ' C C* ' A ' L K C C ' B ' C C* ' B '
for 29): step 04,
C K C A ' B ' C A* ' B ' C K L C C 'A ' L C C* ' A ' K L C C ' B ' L C C* ' B '
for 29): step 14,
C K C A ' B ' C A* ' B ' C K C M C ' L A ' C M C* ' L A ' K C M C ' L B ' C M C* ' L B '
for 29): step 18,
C K C A ' B ' C A* ' B ' C K C A C C ' K AA ' C A C* C* ' K AA ' K C A C C ' K B B ' C A C* C* ' K B B '
for 29): step 07, 08

C K C A ' B ' C A* ' B ' C K C A C C ' K AA ' C A C* C* ' K AA ' K C A C C ' K B B ' C A C* C* ' K B B ' is a tautology (verify it, for example, with the standard tautology calcule), therefore Barbara with assertoric major premisse and apodeitic minor premisse is a **valid syllogism**.

BARBARA (3)

Both premisses problematic

Q a A B , Q a C A √ƒ Q a C B .

C Q a A B C Q A C A Q a C B
for 33),
C K N L a A B N L N a A B C K N L a C A N L N a C A K N L a C B N L N a C B
for 29): step 01,
C K N L K C AB C A* B N L N K C AB C A* B C K N L K C CA C C* A N L N K C CA C C* A K N L K C CB C C* B N L N K C CB C C* B
for 29): step 02,
C K N L K C A ' B ' C A* ' B ' N L N K C A ' B ' C A* ' B ' C K N L K C C 'A ' C C* ' A ' N L N K C C ' A ' C C* ' A ' K N L K C C ' B ' C C* ' B ' N L N K C C ' B ' C C* ' B '
for 29): step 04,
C K N L K C A ' B ' C A* ' B ' M K C A ' B ' C A* ' B ' C K N L K C C 'A ' C C* ' A ' M, K C C ' A ' C C* ' A ' K N L K C C ' B ' C C* ' B ' M K C C ' B ' C C* ' B '
for 29): step 05,06,
C K N K L C A ' B ' L C A* ' B ' K M C A ' B ' M C A* ' B ' C K N K L C C 'A ' L C C* ' A ' K M C C 'A ' M C C* ' A ' K N K L C C ' B ' L C C* ' B ' K M C C ' B ' M C C* ' B '
for 29): step 14,15,
C K N K C M A ' L B ' C M A* ' L B ' K C L A ' M B ' C L A* ' M B ' C K N K C M C ' L A ' C M C* ' L A ' K C L C ' M A ' C L C* ' M A ' K N K C M C ' L B ' C M C* ' L B ' K C L C ' M B ' C L C* ' M B '
for 29): step 18,19,
C K N K C A AA ' K B B ' C A A*A* ' K B B ' K C K AA ' A B B ' C K A*A* ' A B B ' C K N K C A C C ' K AA ' C A C* C* ' K AA ' K C K C C ' A AA 'C K C* C* ' A AA 'K N K C A C C ' K B B ' C A C* C* ' K B B ' K C K C C ' A B B ' C K C* C* ' A B B '
for 29): step 07,08,

C K N K C A AA ' K B B ' C A A*A* ' K B B ' K C K AA ' A B B ' C K A*A* ' A B B ' C K N K C A C C ' K AA ' C A C* C* ' K AA ' K C K C C ' A AA 'C K C* C* ' A AA 'K N K C A C C ' K B B ' C A C* C* ' K B B ' K C K C C ' A B B ' C K C* C* ' A B B ' is a tautology (verify it, for example, with the standard tautology calcule), therefore Barbara with both problematic premisses is a **valid syllogism**.

BARBARA (4)

Problematic major, assertoric minor
Q a A B , a C A √*f* Q a C B .

C Q a A B C A C A Q a C B
for 33),
C K N L a A B N L N a A B C a C A K N L a C B N L N a C B
for 29): step 01,
C K N L K C AB C A* B N L N K C AB C A* B C K C CA C C* A K N L K C CB C C* B N L N K C CB C C* B
for 29): step 02,
C K N L K C A ' B ' C A* ' B ' N L N K C A ' B ' C A* ' B ' C K C C 'A ' C C* ' A ' K N L K C C ' B ' C C* ' B ' N L N K C C ' B ' C C* ' B '
for 29): step 04,
C K N L K C A ' B ' C A* ' B ' M K C A ' B ' C A* ' B ' C KC C 'A ' C C* ' A ' K N L K C C ' B ' C C* ' B ' M K C C ' B ' C C* ' B '
for 29): step 05,06,
C K N K L C A ' B ' L C A* ' B ' K M C A ' B ' M C A* ' B ' C K C C 'A ' C C* ' A ' K N K L C C ' B ' L C C* ' B ' K M C C ' B ' M C C* ' B '
for 29): step 14,15,
C K N K C M A ' L B ' C M A* ' L B ' K C L A ' M B ' C L A* ' M B ' C K C C 'A ' C C* ' A ' K N K C M C ' L B ' C M C* ' L B ' K C L C ' M B ' C L C* ' M B '
for 29): step 18,19,
C K N K C A AA ' K B B ' C A A*A* ' K B B ' K C K AA ' A B B ' C K A*A* ' A B BC K C C 'A ' C C* ' A ' K N K C A C C ' K B B ' C A C* C* ' K B B ' K C K C C ' A B B ' C K C* C* ' A B B '
for 29): step 07,08,

C K N K C A AA ' K B B ' C A A*A* ' K B B ' K C K AA ' A B B ' C K A*A* ' A B BC K C C 'A ' C C* ' A ' K N K C A C C ' K B B ' C A C* C* ' K B B ' K C K C C ' A B B ' C K C* C* ' A B B ' is a tautology (verify it, for example, with the standard tautology calcule), therefore Barbara with problematic major premisse and assertoric minor premisse is a **valid syllogism**.

BARBARA (5)

Assertoric major, problematic minor
a A B , Q a C A √*f* M a C B .

C a A B C Q A C A M a C B
for 33),
C a A B C K N L a C A N L N a C A M a C B
for 29): step 01,
C K C AB C A* B C K N L K C CA C C* A N L N K C CA C C* A M K C CB C C* B
for 29): step 02,
C K C A ' B ' C A* ' B ' C K N L K C C 'A ' C C* ' A ' N L N K C C ' A ' C C* ' A ' M K C C ' B ' C C* ' B '
for 29): step 04,
C K C A ' B ' C A* ' B ' C K N L K C C 'A ' C C* ' A ' M, K C C ' A ' C C* ' A ' M K C C ' B ' C C* ' B '
for 29): step 05,06,
C K C A ' B ' C A* ' B ' C K N K L C C 'A ' L C C* ' A ' K M C C 'A ' M C C* ' A ' K M C C ' B ' M C C* ' B '
for 29): step 14,15,
C K C A ' B ' C A* ' B ' C K N K C M C ' L A ' C M C* ' L A ' K C L C ' A AA ' C L C* ' M A ' K C L C ' M B ' C L C* ' M B '
for 29): step 18,19,
C K C A ' B ' C A* ' B ' C K N K C A C C ' A AA ' C A C* C* ' L A ' K C K C C ' M A ' C K C* C* ' A AA ' K C K C C ' A B B ' C K C* C* ' A B B '
for 29): step 07,08,

C K C A ' B ' C A* ' B ' C K N K C A C C ' A AA ' C A C* C* ' L A ' K C K C C ' M A ' C K C* C* ' A AA ' K C K C C ' A B B ' C K C* C* ' A B B ' is a tautology (verify it, for example, with the standard tautology calcule), therefore Barbara with assertoric major premisse and problematic minor premisse is a **valid syllogism**.

BARBARA (6)

Problematic major, apodeitic minor

Q a A B , L a C A √ƒ Q a C B .

C Q a A B C L A C A Q a C B
for 33),
C K N L a A B N L N a A B C L a C A K N L a C B N L N a C B
for 29): step 01,
C K N L K C AB C A* B N L N K C AB C A* B C L K C CA C C* A K N L K C CB C C* B N L N K C CB C C* B
for 29): step 02,
C K N L K C A ' B ' C A* ' B ' N L N K C A ' B ' C A* ' B ' C L K C C 'A ' C C* ' A ' K N L K C C ' B ' C C* ' B ' N L N K C C ' B ' C C* ' B '
for 29): step 04,
C K N L K C A ' B ' C A* ' B ' M K C A ' B ' C A* ' B ' C L K C C 'A ' C C* ' A ' K N L K C C ' B ' C C* ' B ' M K C C ' B ' C C* ' B '
for 29): step 05,06,
C K N K L C A ' B ' L C A* ' B ' K M C A ' B ' M C A* ' B 'C K L C C 'A ' LC C* ' A ' K N K L C C ' B ' L C C* ' B ' K M C C ' B ' M C C* ' B '
for 29): step 14,15,
C K N K C M A ' L B ' C M A* ' L B ' K C L A ' M B ' C L A* ' M B C K C M C ' L A ' C M C* ' L A ' K N K C M C ' L B ' C M C* ' L B ' K C L C ' M B ' C L C* ' M B '
for 29): step 18,19,
C K N K C A AA ' K B B ' C A A*A* ' K B B ' K C K AA ' A B B ' C K A*A* ' A B B ' C K C A C C ' K AA ' C A C* C* ' K AA ' K N K C A C C ' K B B ' C A C* C* ' K B B ' K C K C C ' A B B ' C K C* C* ' A B B '
for 29): step 07,08,

C K N K C A AA ' K B B ' C A A*A* ' K B B ' K C K AA ' A B B ' C K A*A* ' A B B ' C K C A C C ' K AA ' C A C* C* ' K AA ' K N K C A C C ' K B B ' C A C* C* ' K B B ' K C K C C ' A B B ' C K C* C* ' A B B " is a tautology (verify it, for example, with the standard tautology calcule), therefore Barbara with problematic major premisse and apodeitic minor premisse is a **valid syllogism**.

BARBARA (7)

Apodeitic major, problematic minor
L a A B , Q a C A √*f* M a C B .

C L a A B C Q A C A M a C B
for 33),
C L a A B C K N L a C A N L N a C A M a C B
for 29): step 01,
C L K C AB C A* B C K N L K C CA C C* A N L N K C CA C C* A M K C CB C C* B
for 29): step 02,
C L K C A ' B ' C A* ' B ' C K N L K C C 'A ' C C* ' A ' N L N K C C 'A ' C C* ' A ' M K C C ' B ' C C* ' B '
for 29): step 04,
C L K C A ' B ' C A* ' B ' C K N L K C C 'A ' C C* ' A ' M K C C 'A ' C C* ' A ' M K C C ' B ' C C* ' B '
for 29): step 05,06,
C K LC A ' B ' LC A* ' B ' C K N K LC C 'A ' LC C* ' A ' K M C C 'A ' M C C* ' A ' K M C C ' B ' M C C* ' B '
for 29): step 14,15,
C K C M A ' L B ' C M A* ' L B ' C K N K C M C ' L A ' C M C* ' L A ' K C L C ' M A ' C L C* ' M A ' K C L C ' M B ' C L C* ' M B '
for 29): step 18,19,
C K C A AA ' K B B ' C A A*A* ' K B B ' C K N K C A C C ' K AA ' C A C* C* ' K AA ' K C K C C ' A AA ' C K C* C* ' A AA ' K C K C C ' A B B ' C K C* C* ' A B B '
for 29): step 07,08,

C K C A AA ' K B B ' C A A*A* ' K B B ' C K N K C A C C ' K AA ' C A C* C* ' K AA ' K C K C C ' A AA ' C K C* C* ' A AA ' K C K C C ' A B B ' C K C* C* ' A B B ' is a tautology (verify it, for example, with the standard tautology calcule), therefore Barbara with apodeitic major premisse and problematic minor premisse is a **valid syllogism**.